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TEMPERATURE AND MOISTURE DISTRIBUTIONS DURING CONTACT DRYING OF A MOIST POROUS SHEET

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Abstract—Using Luikov's set of differential equations, the drying of a layer of moist material in contact with a hot plate is investigated. In this journal the same problem is studied by Bruin [1] with the simplifying assumption that the moisture movement under influence of moisture potential gradient is negligible. The present analysis is based on an exact analytical solution without the mentioned restriction. The influence of dimensionless parameters on the temperature and moisture potential distributions is illustrated by numerical examples.

NOMENCLATURE

Dimensionless criteria

$Fo = a_q\tau/D^2$, Fourier number;
$Lu = a_m/a_q$, Luikov number;
$Bi_q = \alpha_q D/\lambda_q$, Biot number for heat transfer;
$Bi_m = \alpha_m D/\lambda_m$, Biot number for mass transfer;
$Ko = \dfrac{rc_m(\theta_0-\theta_*)}{c_q(t_s-t_0)}$, Kossovitch number;
$Pn = \dfrac{\delta(t_s-t_0)}{c_m(\theta_0-\theta_*)}$, Posnov number;
ε, phase change criterion;
$T(X, Fo) = \dfrac{t-t_0}{t_s-t_0}$, dimensionless temperature;
$\theta(X, Fo) = \dfrac{\theta_0-\theta}{\theta_0-\theta_*}$, dimensionless moisture transfer potential;
$X = x/D$, dimensionless coordinate;

where

x, coordinate perpendicular to the surface [L];
t, temperature [°C];
θ, moisture potential [°M];
τ, time [T];
a_q, thermal diffusivity coefficient [L^2T^{-1}];
a_m, diffusion coefficient of moisture in the material [L^2T^{-1}];
r, specific heat of evaporation [L^2T^{-2}];
δ, thermal gradient coefficient [$°C^{-1}$];
c_m, specific isothermal mass capacity of the material [$°M^{-1}$];
c_q, specific heat capacity of the material [$L^2T^{-2}°C^{-1}$];
α_q, heat-transfer coefficient [$mT^{-3}°C^{-1}$];
α_m, mass-transfer coefficient [$mL^{-2}T^{-1}°M^{-1}$];
λ_q, thermal conductivity [$mLT^{-3}°C^{-1}$];
λ_m, moisture conductivity [$mL^{-1}T^{-1}°M^{-1}$];
ϕ_q, heat flux [mT^{-3}];
D, thickness of the layer of moist material [L];
$Ki_q = \dfrac{D\phi_q}{\lambda_q(t_s-t_0)}$, dimensionless heat flux.

Subscripts

0, initial;
s, surroundings;
*, in equilibrium with surrounding air.

1. INTRODUCTION

AN EXACT computation of temperature and moisture distribution when drying porous bodies can be accomplished through numerical solution of the well-known Luikov's system of coupled partial differential equations. Such an approach has not found wide application chiefly because there is not enough data about the dependance of the heat-transfer parameters on moisture and temperature. The information about the temperature and moisture distributions, obtained in this way is true, but it is valid only for concrete material under given conditions of drying.

Taking an average for the heat-transfer parameters, the system, mentioned above, can be linearized [2]. In this case the results obtained and the results expected do not coincide so well, but on the other hand it is possible to make a quantitative analysis of the influence of the nondimensional parameters on temperature and moisture changes. The results obtained through such an approach are universal. That is why many investigations were made on the base of the linearized system [3].

A study of temperature and moisture distributions during contact drying of a sheet of moist material was presented by Bruin [1]. Having in mind the great practical importance of this paper, it was discussed in detail in [4].

Because of mathematical difficulties, arising from assymmetry of boundary conditions Bruin did not succeed in finding an exact analytical solution of the problem. That is why he used the simplifying assumption of Makavozov [5–6], that is that the moisture potential gradient does not influence the movement of moisture, assuming that $Lu(\partial^2\theta/\partial x^2) = 0$.

This simplification of the linearized system is inadmissible. It leads to similar distributions of moisture and temperature potentials, thus reducing Luikov's system to a single partial differential equation. That is why the Laplace image of the solutions contains only two constants, when there are four boundary conditions for their satisfaction. That was the reason why Bruin next united incorrectly the boundary conditions thus Bi_m being excluded. But Bi_m is a parameter with immense influence on the moisture distribution and hence on the temperature distribution.

Our opinion is that Bruin's graphics give not only a quantitively but also a qualitively untrue picture of temperature and moisture fields. To substantiate this opinion in the present work we give an exact analytical solution of Bruin's problem, on the base of which the examples in [1] (Fig. 2a, b, c) are calculated. It was not difficult to obtain such an exact solution, because it is contained in the recently published general solution [7].

2. STATEMENT AND SOLUTION OF THE PROBLEM

In [1] (see Fig. 1, p. 46) Bruin analysed the contact drying of a moist porous sheet on a hot plate.

Temperature and moisture distributions is described by Luikov's system

$$\frac{\partial T(X,Fo)}{\partial Fo} = \frac{\partial^2 T(X,Fo)}{\partial X^2} - \varepsilon Ko\frac{\partial\theta(X,Fo)}{\partial Fo} \tag{1}$$

$$\frac{\partial\theta(X,Fo)}{\partial Fo} = Lu\frac{\partial^2\theta(X,Fo)}{\partial X^2} - LuPn\frac{\partial^2 T(X,Fo)}{\partial X^2}. \tag{2}$$

Equations (1) and (2) are subjected to the following conditions:

Initial condition:

$$T(X,0) = 0, \qquad \theta(X,0) = 0. \tag{3}$$

Heat flux through the hot plate:

$$\frac{\partial T(0,Fo)}{\partial X} = -Ki_q. \tag{4}$$

Mass balance on the surface of the hot plate

$$\frac{\partial\theta(0,Fo)}{\partial X} - Pn\frac{\partial T(0,Fo)}{\partial X} = 0. \tag{5}$$

Heat balance on the free surface:

$$\frac{\partial T(1,Fo)}{\partial X} - Bi_q[1-T(1,Fo)] + (1-\varepsilon)KoLuBi_m[1-\theta(1,Fo)] = 0. \tag{6}$$

Mass balance on the free surface:

$$-\frac{\partial\theta(1,Fo)}{\partial X} + Pn\frac{\partial T(1,Fo)}{\partial X} + Bi_m[1-\theta(1,Fo)] = 0. \tag{7}$$

The exact analytical solution of the system (1)–(2) under the initial condition (3) and the boundary conditions (4)–(7) is obtained in Appendix 1 and has the form:

$$T(X,Fo) = 1 + Ki_q\left(1+\frac{1}{Bi_q}-X\right) - \sum_{i=1}^{\infty} A_i e^{-\mu_i^2 Fo} \times \sum_{j=1}^{2}(-1)^j(\vartheta_j^2-1)b_j(\mu_i)\cos(\vartheta_{3-j}\mu_i X) \tag{8}$$

$$\theta(X,Fo) = 1 + PnKi_q(1-X) + Pn\sum_{i=1}^{\infty} A_i e^{-\mu_i^2 Fo} \times \sum_{j=1}^{2}(-1)^j b_j(\mu_i)\cos(\vartheta_{3-j}\mu_i X) \tag{9}$$

where

$$\vartheta_j^2 = \frac{1}{2}\left\{1+\varepsilon KoPn+\frac{1}{Lu}+(-1)^j \times\sqrt{\left[\left(1+\varepsilon KoPn+\frac{1}{Lu}\right)^2-\frac{4}{Lu}\right]}\right\} \tag{10}$$

$$A_i = \frac{2}{\mu_i^2}\left\{\frac{Ki_q}{\vartheta_2^2-\vartheta_1^2}[b_2(\mu_i)c_2 - b_1(\mu_i)c_1] + \left\{\left[1-(1-\varepsilon)KoLu\frac{Bi_m}{Bi_q}\right]b_1(\mu_i) + \frac{\vartheta_2^2-1}{Pn}a_1(\mu_i)\right\}b_2(\mu_i)\right\} \times\{b_2^2(\mu_i)c_2d_1(\mu_i) - b_1^2(\mu_i)c_1d_2(\mu_i)\}^{-1} \tag{11}$$

$$a_j(\mu_i) = \left[1+(1-\vartheta_j^2)\frac{1-\varepsilon}{\varepsilon}Lu\frac{Bi_m}{Bi_q}\right] \times\cos(\vartheta_j\mu_i) - \frac{\vartheta_j\mu_i}{Bi_q}\sin(\vartheta_j\mu_i) \tag{12}$$

$$b_j(\mu_i) = \cos(\vartheta_j\mu_i) - \frac{\mu_i}{LuBi_m\vartheta_j}\sin(\vartheta_j\mu_i) \tag{13}$$

$$c_j = (\vartheta_j^2-1)\left(\frac{1}{Bi_q}-\frac{1}{LuBi_m}\right) - \frac{KoPn}{Bi_q} \tag{14}$$

$$d_j(\mu_i) = 1 + \frac{\sin(\vartheta_j\mu_i)}{\vartheta_j\mu_i}\cos(\vartheta_j\mu_i). \tag{15}$$

The μ_i are the positive roots of the characteristic equation $f(\mu_i) = 0$, where:

$$f(\mu_i) = (1-\vartheta_2^2)a_1(\mu_i)b_2(\mu_i) - (1-\vartheta_1^2)a_2(\mu_i)b_1(\mu_i). \quad (16)$$

The roots of this equation were calculated with an accuracy of seven correct symbols using the method of dividing the interval into halves. The comparison of the signs of $F(\mu)$ at the ends of the consequent intervals with length h gives the interval, where a root is supposed to exist. The presence of different signs is an indication for the existence of a root in the corresponding interval.

A substantial defect of this simple method is the possibility of omitting roots when choosing too large a step h. After a numerical experiment we chose as a most appropriate step $h = 0{\cdot}02$, but it does not guarantee the omission of roots.

In Table 1 are given the first twenty-five roots of equation (16) with five correct symbols after the decimal point for some of the cases analysed. The utility of publishing of detailed tables of roots is doubted because their calculation each time is considered to be more convenient than their being introduced as input data in a computer. Table 1 is supposed to serve as an appliance to those investigators, who would wish to program our solution.

Table 1. Roots of equation (16)
$Lu = 0{\cdot}4$, $Pn = 0{\cdot}6$, $Ko = 5$, $Bi_q = 5$

/i	$\varepsilon = 0{\cdot}2$			
	$Bi_m = 1$	$Bi_m = 2{\cdot}5$	$Bi_m = 5$	$Bi_m = 10$
1	0·51519	0·63831	0·69360	0·72335
2	1·34300	3·61562	3·70757	3·78244
3	1·79173	4·36375	4·73523	6·98836
4	3·52453	5·11397	4·91463	10·32595
5	4·22759	6·91995	6·94944	13·72017
6	5·16807	7·71052	7·87753	17·13578
7	6·89770	8·55753	8·48721	20·56010
8	7·62573	10·31443	10·31890	23·98821
9	8·58915	11·22749	11·34959	27·41802
10	10·31135	11·99226	11·94316	30·84856
11	11·16115	13·73204	13·72768	32·60510
12	12·01553	14·79911	14·90019	32·93866
13	13·73489	15·42423	15·38302	34·27936
14	14·74381	17·15704	17·14951	36·01884
15	15·44358	18·39636	18·48794	36·57647
16	17·16181	18·85436	18·81354	37·71014
17	18·34788	20·58487	20·57632	39·44282
18	18·87225	22·00971	22·11613	40·20748
19	20·59016	22·28135	22·21891	41·14069
20	21·96365	24·01393	24·00522	42·87024
21	22·30093	27·44357	27·43504	43·83796
22	24·01923	30·87351	30·86529	44·57075
23	25·58828	32·59780	32·60072	46·29918
24	25·72771	32·85158	32·87995	47·46984
25	27·44870	34·30361	34·29571	47·99976

Table 1. (*continued*)

/i	$\varepsilon = 0{\cdot}4$			
	$Bi_m = 1$	$Bi_m = 2{\cdot}5$	$Bi_m = 5$	$Bi_m = 10$
1	0·49959	0·60870	0·65670	0·68259
2	1·34721	3·27246	3·34800	3·43193
3	1·69665	6·26861	6·28007	6·30273
4	3·20967	7·84823	7·89435	7·94972
5	6·26174	8·39195	8·49463	8·73540
6	7·81258	9·34485	9·33168	9·29293
7	8·33908	10·91057	10·92161	10·94124
8	9·35159	14·00474	14·00620	14·00918
9	10·90351	15·56504	15·57705	15·59657
10	12·27194	16·26407	16·32291	16·44665
11	12·44475	17·10395	17·09854	17·08507
12	14·00388	18·66226	18·66573	18·67237
13	15·55698	21·76506	21·76527	21·76570
14	16·22997	23·32121	23·32580	23·33407
15	17·10688	24·23847	24·28127	24·36948
16	18·66012	24·86806	24·86398	24·85383
17	21·76494	26·42478	26·42631	26·42932
18	23·31830	27·98513	27·99496	28·00944
19	24·21321	28·22910	28·25572	28·31408
20	24·87026	29·52982	29·52966	29·52932
21	26·42386	31·08479	31·08702	31·09123
22	27·97818	32·24241	32·27725	32·35056
23	28·21419	32·63355	32·62892	32·61615
24	29·52991	34·19026	34·19105	34·19263
25	31·08341	35·74585	35·74950	35·75587

/i	$\varepsilon = 0{\cdot}8$			
	$Bi_m = 1$	$Bi_m = 2{\cdot}5$	$Bi_m = 5$	$Bi_m = 10$
1	0·47217	0·56011	0·59744	0·61754
2	1·34923	2·89373	2·97822	3·07664
3	1·56302	4·20726	4·29142	4·37686
4	2·82690	5·29285	6·85339	6·92607
5	4·13418	5·38607	8·19809	8·26442
6	5·11142	6·81454	10·84592	10·89357
7	5·45586	8·15659	12·19444	12·24075
8	6·79104	10·82354	13·54706	13·59208
9	8·12897	12·16937	14·15944	14·26027
10	9·47411	13·51774	14·86293	14·89580
11	9·52488	14·10957	16·21477	16·24946
12	10·81071	14·84915	17·56219	17·59594
13	12·15381	16·19714	18·77126	20·27114
14	13·49775	17·54347	18·86232	21·61777
15	14·08092	18·70734	20·24370	22·97085
16	14·84166	18·87321	21·59084	23·38183
17	16·18652	20·23031	22·94318	24·29856
18	17·53165	21·57676	23·32264	25·64652
19	18·67239	22·92579	24·27626	26·99378
20	18·87654	23·29531	25·62412	27·99579
21	20·22241	24·26587	26·97161	28·32543
22	21·56814	25·61279	27·94214	29·67876
23	22·91410	26·95961	28·30859	31·02550
24	23·27993	27·91393	29·65963	32·38121
25	24·25987	28·30183	31·00663	32·58617

From physical considerations it is clear that for small values of the nondimensional time ($Fo < 0{\cdot}05$) and close to the initial surface ($X = 0$) the results obtained through solutions (8) and (9) are supposed to coincide with those, obtained from the solutions for a semispace, subjected to the same influences. Such solutions are given in Appendix 2.

$$T(X,Fo) = \frac{2(Fo)^{\frac{1}{2}}Ki_q}{\vartheta_2^2-\vartheta_1^2}\sum_{j=1}^{2}(-1)^j \left[-\frac{1}{\pi}\vartheta_j(\vartheta_{3-j}^2-1)\exp\left(-\left[\frac{\vartheta_j X}{2(Fo)^{\frac{1}{2}}}\right]\right)^2 + \frac{X}{2(Fo)^{\frac{1}{2}}}\left(\frac{1}{Lu}-\vartheta_j^2\right)\mathrm{erfc}\left(\frac{\vartheta_j X}{2(Fo)^{\frac{1}{2}}}\right)\right] \quad (17)$$

$$\theta(X,Fo) = Pn\frac{2(Fo)^{\frac{1}{2}}Ki_q}{\vartheta_2^2-\vartheta_1^2}\sum_{j=1}^{2}(-1)^j \left[\frac{1}{\pi}\vartheta_j\exp\left(-\left[\frac{\vartheta_j X}{2(Fo)^{\frac{1}{2}}}\right]\right)^2 - \frac{X}{2(Fo)^{\frac{1}{2}}}\vartheta_j^2\,\mathrm{erfc}\left(\frac{\vartheta_j X}{2(Fo)^{\frac{1}{2}}}\right)\right]. \quad (18)$$

The possibility of omitting roots in calculating μ_i made us program the solutions (17)–(18). The correctness of the results given in the present paper was controlled through them.

3. DISCUSSION OF RESULTS AND NUMERICAL EXAMPLES

The temperature and moisture distributions were calculated in terms of following set of variables [1]:

$Lu = 0{\cdot}4, 0{\cdot}02;\quad \varepsilon = 0{\cdot}2, 0{\cdot}4, 0{\cdot}8, 1{\cdot}0$
$Pn = 0{\cdot}6;\quad Ko = 5{\cdot}0;\quad Ki_q = 0{\cdot}9;\quad Bi_q = 2{\cdot}5;$
$Fo = 0{\cdot}05, 0{\cdot}1, 0{\cdot}2, 0{\cdot}4, 0{\cdot}8, 1{\cdot}6, 3{\cdot}2, 6{\cdot}4.$

From the thermodynamical point of view the heat and mass transfer is not equiprobable for arbitrary values of Lu and $\varepsilon KoPn$ [2]. For small values of Lu values of $\varepsilon KoPn < 0{\cdot}3$ are supposed to be more probable while if $Lu > 0{\cdot}3$ then $\varepsilon KoPn > 0{\cdot}4$.

In our examples $\varepsilon KoPn \geqslant 0{\cdot}6$ and, consequently $Lu = 0{\cdot}02$ is more or less improbable. Aiming the comparison of our results with the ones, given in [1], we calculated such an example too.

In contrast to [1] for Fo the value 0·2 is added and the value of the phase change criterion $\varepsilon = 0{\cdot}6$ is taken to be 0·4. This change is made because from known experimental results ε varies in the intervals 0·0–0·4 and 0·8–1·0. In [2] it is stated that the experiments of Polonskaia and Lebedev show that for a gypsom plate

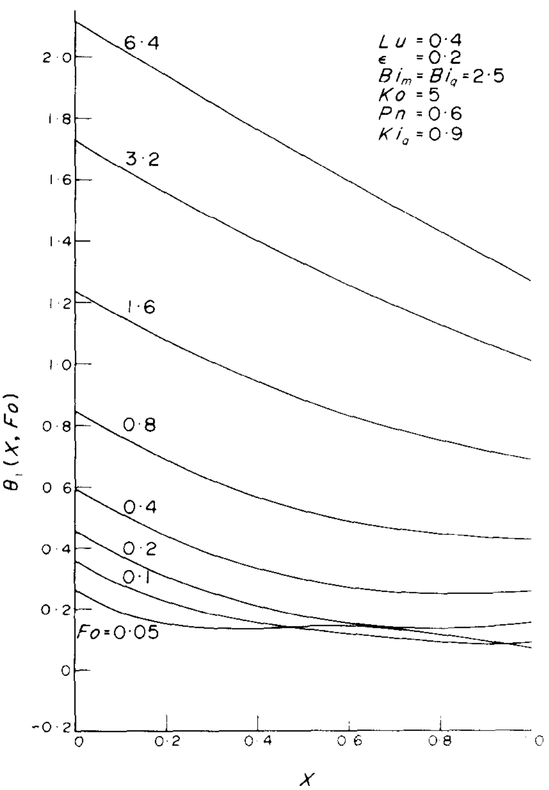

FIG. 1. Dimensionless temperature distributions during contact drying for $Lu = 0{\cdot}4$, $\varepsilon = 0{\cdot}2$, $Bi_m = Bi_q = 2{\cdot}5$, $Ko = 5$, $Pn = 0{\cdot}6$ and $Ki_q = 0{\cdot}9$.

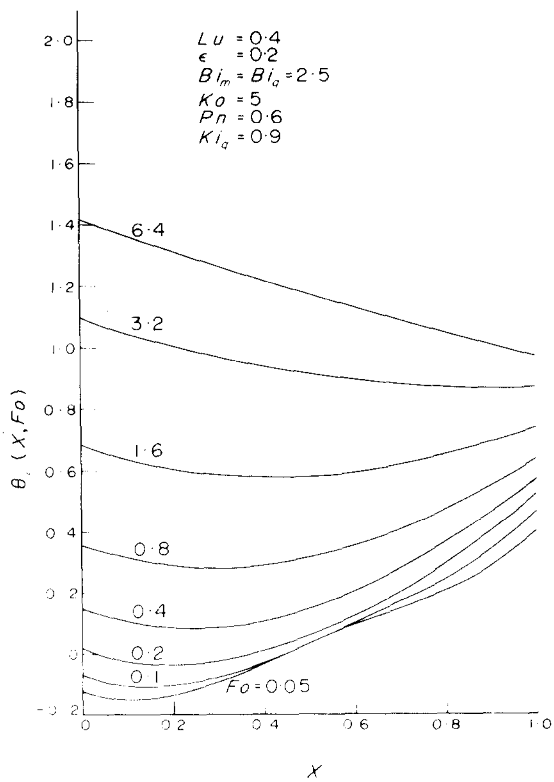

FIG. 2. Dimensionless moisture transfer potential distributions for $Lu = 0{\cdot}4$, $\varepsilon = 0{\cdot}2$, $Bi_q = Bi_m = 2{\cdot}5$, $Ko = 5$, $Pn = 0{\cdot}6$ and $Ki_q = 0{\cdot}9$.

$\varepsilon = 0{\cdot}045$; for clay 0·75–1·0; for sand 0·2–0·4 and for wood 0·09–0·2.

The heat- and mass-transfer criterions of Biot in real conditions depend on each other. This interrelation is established through the balances of heat and mass. The change of Bi_q implies a change of Bi_m and their numerical values are of approximately the same order [2].

Taking into account the fact that Bi_m is not present in the solutions, given in [1], in our fundamental examples we accepted it to be equal to $Bi_q = 2{\cdot}5$. In order to investigate the influence of Bi_m, its values were varied according to examples, given in [2], as follows:

$$Bi_m = 1{\cdot}0, 2{\cdot}5, 5{\cdot}0, 10{\cdot}0.$$

On the basis of solutions (8)–(16) an ALGOL program for computation of $T(X, Fo)$ and $\theta(X, Fo)$ was prepared. Although Table 1 gives only the first twenty-five roots in our program this number is determined automatically so that the prescribed accuracy for the computation of the temperature and moisture distributions is guaranteed.

The latter two were computed with an accuracy of five correct symbols, which to some degree was useless, because such an accuracy is not necessary when drawing the graphics.

Figures 1 and 2 represent the fundamental case: $Lu = 0{\cdot}4$, $\varepsilon = 0{\cdot}2$, $Bi_m = 2{\cdot}5$. The distributions, shown in these figures, are represented in [1] through only one graphic because there the potentials are related linearly with a proportionality factor $(1+\varepsilon LuKoPn)/(LuPn) = 5{\cdot}166\ldots$ for the case under consideration; this means that $T(X, Fo) = 5{\cdot}166\theta(X, Fo)$. This linear relation is not corroborated by our results.

In [1] it is pointed out that the possibilities of comparison of the analytical results with experimental data in literature are very restricted because of the scarcity of such data. That is why Bruin announces about his own experiments, where the experimental curves show an inflection point in the moisture potential distribution. This phenomenon is predicted by our solutions and can be found in Fig. 2 for values of $Fo = 0{\cdot}05$ and $0{\cdot}1$.

For small values of the nondimensional time the distribution of the potentials is particularly unstable

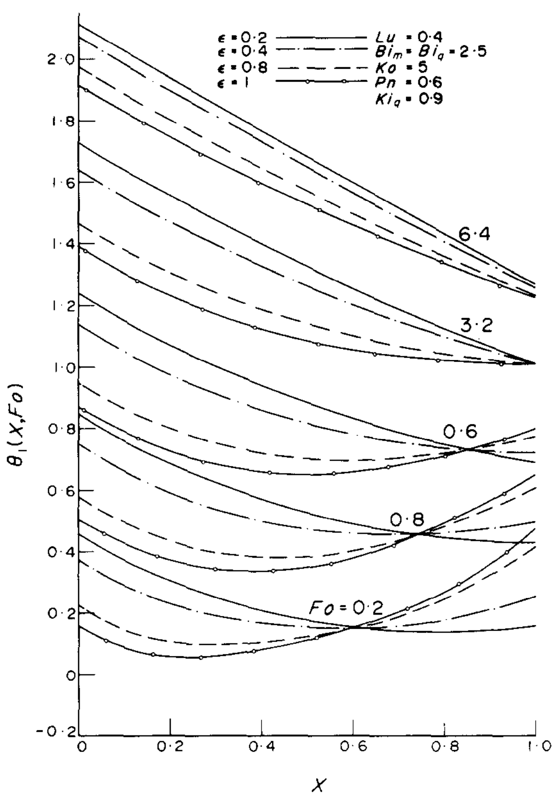

FIG. 3. Influence of the phase change criterion ($\varepsilon = 0{\cdot}2$, $0{\cdot}4$, $0{\cdot}8$ and 1) on the temperature distribution for $Lu = 0{\cdot}4$, $\varepsilon = 0{\cdot}2$, $Bi_m = Bi_q = 2{\cdot}5$, $Ko = 5$, $Pn = 0{\cdot}6$ and $Ki_q = 0{\cdot}9$.

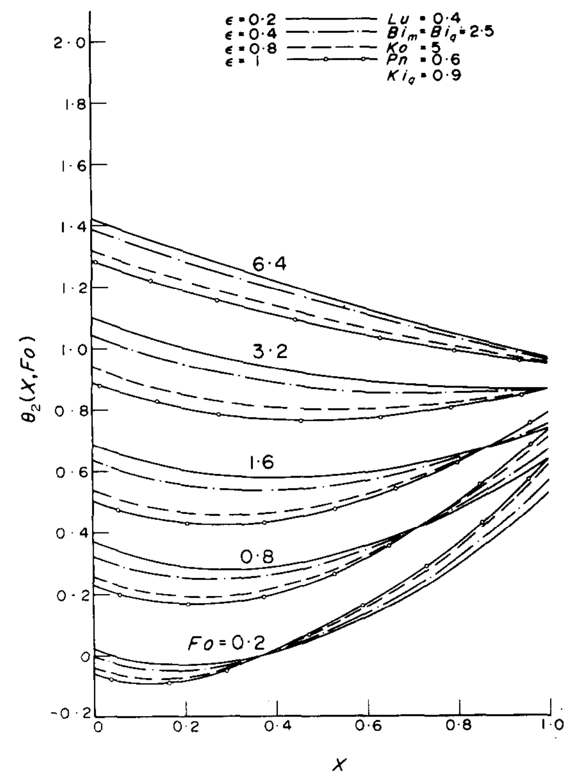

FIG. 4. Influence of the phase change criterion ($\varepsilon = 0{\cdot}2$, $0{\cdot}4$, $0{\cdot}8$ and 1) on the moisture transfer potential distributions for $Lu = 0{\cdot}4$, $\varepsilon = 0{\cdot}2$, $Bi_q = Bi_m = 2{\cdot}5$, $Ko = 5$, $Pn = 0{\cdot}6$ and $Ki_q = 0{\cdot}9$.

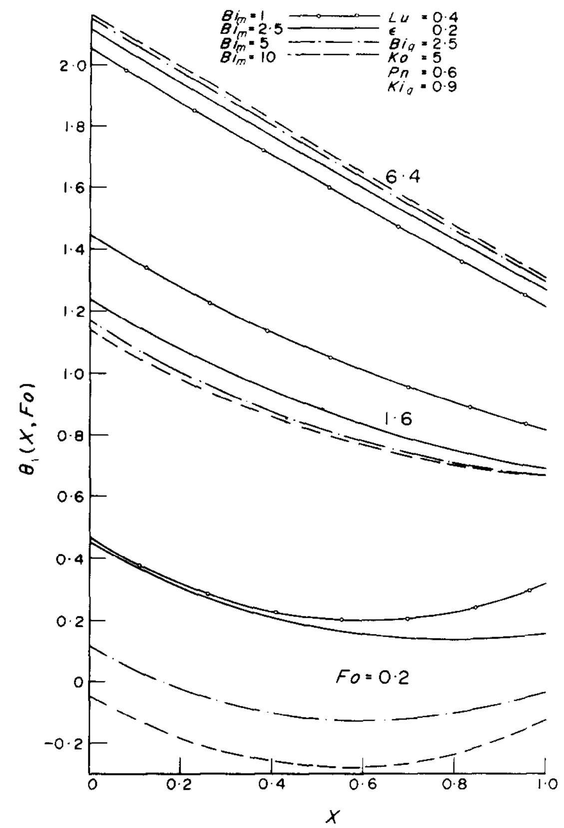

FIG. 5. Influence of Biot number for mass transfer ($Bi_m = 1$, 2·5, 5 and 10) on the temperature distributions for $Lu = 0{\cdot}4$, $\varepsilon = 0{\cdot}2$, $Bi_q = 2{\cdot}5$, $Ko = 5$, $Pn = 0{\cdot}6$ and $Ki_q = 0{\cdot}9$.

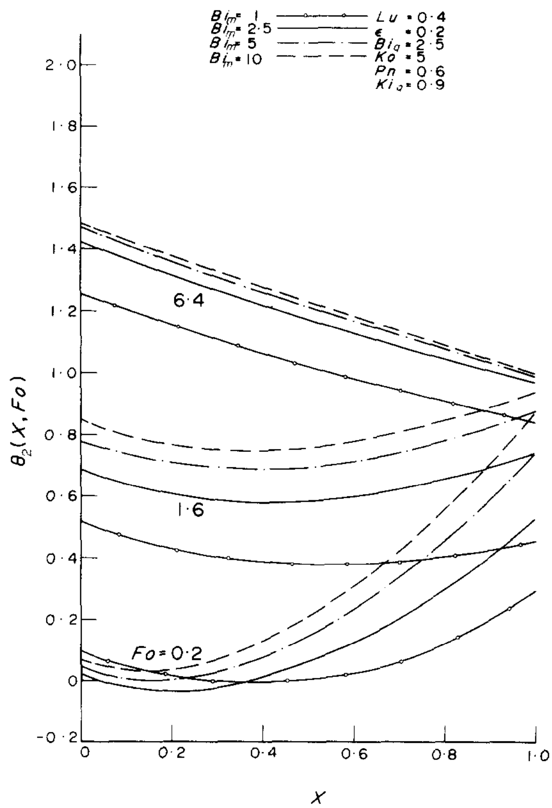

FIG. 6. Influence of Biot number for mass transfer ($Bi_m = 1$, 2·5, 5 and 10) on the moisture transfer potential distributions for $Lu = 0{\cdot}4$, $\varepsilon = 0{\cdot}2$, $Bi_q = Bi_m = 2{\cdot}5$, $Ko = 5$, $Pn = 0{\cdot}6$ and $Ki_q = 0{\cdot}9$.

and is characterized not only by a moisture, but also by a temperature wave. The latter can be seen from the graphics for values of $Fo = 0{\cdot}05$ and $Fo = 0{\cdot}10$, which are shown on Fig. 1.

Figures 3 and 4 represent the influence of the phase change criterion. In [1] this influence is shown for only one value of Fo and the following conclusion is drawn: "a low value of ε gives higher temperatures in the material, because less heat is needed for evaporation of moisture". From Fig. 3 it is obvious that this conclusion is true only for large values of Fo. For small values of the nondimensional time the conclusion is correct only for this region of the plate which is disposed close to the hot plate, while for the area near the free surface the effect is just the opposite. It is interesting to note, that the change described occurs in a point, which moves in time towards the free surface and in our case reaches it when $Fo = 3{\cdot}2$.

The moisture potential shown in Fig. 4, behaves in an analogous way and for large values of the nondimensional time ($Fo > 0{\cdot}8$) the point of change is to be found for both distributions an approximately equal distance from the free surface.

Figures 5 and 6 represent the immense influence of Bi_m both on temperature and moisture fields. Exceptionally interesting is the change of place of the temperature lines. In order to realize the prescribed large values of Bi_m for small values of Fo one can observe temperatures even lower in comparison to the initial ones. This is not contradictory to logics, because the heat income in the beginning is not enough and the prescribed evaporation can occur only at the expense of cooling of the capillary-porous plate. On the whole Bi_m influences the process chiefly in the beginning.

Figures 7 and 8 represent results for $Lu = 0{\cdot}02$ and $Bi_m = 2{\cdot}5$ and 5·0. These figures once again show that there is not similarity between the two distributions. They confirm the well-known fact [2], that Luikov number substantially affects the distributions. For small values of Lu the temperature field develops much more rapidly in comparison with the moisture one.

The present investigation shows that quite a complex mechanism is hiding behind the exterior simplicity of the process of drying. Its investigation should be performed only on the basis of Luikov's system without

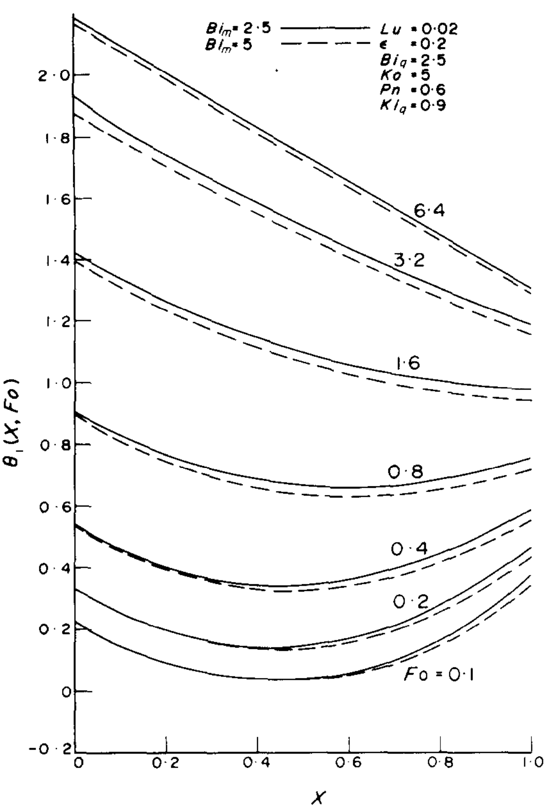

FIG. 7. Influence of Biot number for mass transfer ($Bi_m = 2{\cdot}5$ and 5) on the temperature distribution for small value of Luikov number ($Lu = 0{\cdot}02$) and $\varepsilon = 0{\cdot}2$, $Bi_q = 2{\cdot}5$, $Ko = 5$, $Pn = 0{\cdot}6$ and $Ki_q = 0{\cdot}9$.

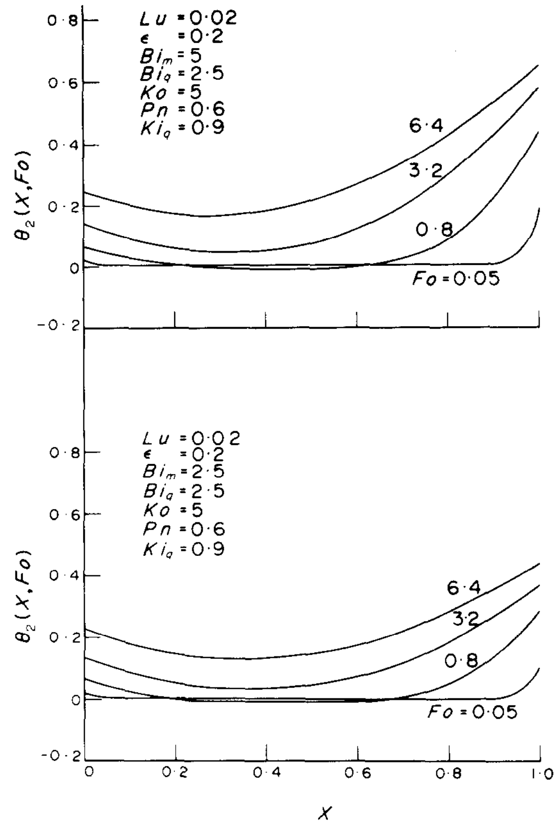

FIG. 8. Influence of Biot number for mass transfer ($Bi_m = 2{\cdot}5$ and 5) on the moisture transfer potential distributions for small value of Luikov number ($Lu = 0{\cdot}02$) and $\varepsilon = 0{\cdot}2$, $Bi_q = 2{\cdot}5$, $Ko = 5$, $Pn = 0{\cdot}6$ and $Ki_q = 0{\cdot}9$.

any simplifications of the latter, because the simplifying assumption of Makavozov that the moisture movement under the influence of moisture potential gradient is negligible, leads to qualitatively untrue picture of temperature and moisture distributions.

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APPENDIX A

Subject of this Appendix is the obtaining of an exact analytical solution of Luikov's system

$$\frac{\partial T(X,Fo)}{\partial Fo} = \frac{\partial^2 T(X,Fo)}{\partial X^2} - \varepsilon Ko \frac{\partial \theta(X,Fo)}{\partial Fo} \tag{1A}$$

$$\frac{\partial \theta(X,Fo)}{\partial Fo} = Lu \frac{\partial^2 \theta(X,Fo)}{\partial X^2} - Lu Pn \frac{\partial^2 T(X,Fo)}{\partial X^2} \tag{2A}$$

under the following initial and boundary conditions:

$$T(X,0) = \theta(X,0) = 0 \tag{3A}$$

$$\frac{\partial T(0,Fo)}{\partial X} = -Ki_q \tag{4A}$$

$$\frac{\partial \theta(0,Fo)}{\partial X} = -PnKi_q \tag{5A}$$

$$T(1,Fo) - (1-\varepsilon)KoLu\frac{Bi_m}{Bi_q}\theta(1,Fo) + \frac{1}{Bi_q}\frac{\partial T(1,Fo)}{\partial X} = 1-(1-\varepsilon)KoLu\frac{Bi_m}{Bi_q} \tag{6A}$$

$$\theta(1,Fo) - \frac{Pn}{Bi_m}\frac{\partial T(1,Fo)}{\partial X} + \frac{1}{Bi_m}\frac{\partial \theta(1,Fo)}{\partial X} = 1. \tag{7A}$$

This problem is a particular case of the recently published general solution [7]. The numbers in curly brackets refer to the equations of [7].

The comparison of equations (4A) and (5A) with equation {37} shows that for the case under consideration:

$$A(0) = 1; \quad B(0) = 0; \qquad \Omega_1(0, Fo) = -Ki_q; \quad \Omega_2(0, Fo) = -PnKi_q. \tag{8A}$$

From an analogous confrontation of the boundary conditions (6A) and (7A) with equation {36} it follows that:

$$K_{11} = 1; \quad K_{12} = -(1-\varepsilon)KoLu\frac{Bi_m}{Bi_q};$$

$$K_{13} = \frac{1}{Bi_q}; \quad K_{14} = 0;$$

$$K_{21} = 0; \quad K_{22} = 1; \quad K_{23} = -\frac{Pn}{Bi_m}; \quad K_{24} = \frac{1}{Bi_m};$$

$$\Omega_1(1, Fo) = 1-(1-\varepsilon)KoLu\frac{Bi_m}{Bi_q}; \quad \Omega_2(1, Fo) = 1. \tag{9A}$$

With the help of (9A) and {45}, and having in mind that $(\vartheta_1^2-1)(\vartheta_2^2-1) = -\varepsilon KoPn$, one can calculate the coefficients:

$$L_{11} = (\vartheta_2^2-1)1+(1-\vartheta_1^2)\frac{1-\varepsilon}{\varepsilon}Lu\frac{Bi_m}{Bi_q};$$

$$L_{12} = -(\vartheta_1^2-1)\left[1+(1-\vartheta_2^2)\frac{1-\varepsilon}{\varepsilon}Lu\frac{Bi_m}{Bi_q}\right];$$

$$L_{13} = \frac{\vartheta_2^2-1}{Bi_q}; \quad L_{14} = -\frac{\vartheta_1^2-1}{Bi_q}; \quad L_{21} = -Pn; \quad L_{22} = Pn;$$

$$L_{23} = -Pn\frac{\vartheta_2^2}{Bi_m}; \quad L_{24} = Pn\frac{\vartheta_1^2}{Bi_m}. \tag{10A}$$

The required solution for $T(X, Fo)$ and $\theta(X, Fo)$ are given by equations {41} and {42}, which can be written in the form:

$$T(X, Fo) = \frac{1}{\vartheta_2^2-\vartheta_1^2}\sum_{j=1}^{2}(-1)^j(1-\vartheta_{3-j}^2)Z_j(X, Fo) \tag{11A}$$

$$\theta(X, Fo) = \frac{Pn}{\vartheta_2^2-\vartheta_1^2}\sum_{j=1}^{2}(-1)^j Z_j(X, Fo) \tag{12A}$$

where according to equation {40}:

$$\vartheta_j^2 = \frac{1}{2}\left[1+\varepsilon KoPn+\frac{1}{Lu}+(-1)^j \times\left(\left(1+\varepsilon KoPn+\frac{1}{Lu}\right)^2-\frac{4}{Lu}\right)^{\frac{1}{2}}\right]; \quad j = 1, 2. \tag{13A}$$

The potentials $Z_j(X, Fo)$ are determined from the system {39}, {44} and {46}. For the case under discussion, having in mind (3A), (8A), (9A) and (10A), one gets:

$$\vartheta_j^2\frac{\partial Z_j(X, Fo)}{\partial Fo} = \frac{\partial^2 Z_j(X, Fo)}{\partial X^2}, \quad j = 1, 2 \tag{14A}$$

$$Z_j(X, 0) = 0, \quad j = 1, 2 \tag{15A}$$

$$\frac{\partial Z_j(0, Fo)}{\partial X} = -Ki_q\vartheta_j^2, \quad j = 1, 2 \tag{16A}$$

$$\sum_{j=1}^{2}(-1)^j(1-\vartheta_{3-j}^2) \times\left\{\left[1+(1-\vartheta_j^2)\frac{1-\varepsilon}{\varepsilon}Lu\frac{Bi_m}{Bi_q}\right]Z_j(1, Fo)+\frac{1}{Bi_q}\frac{\partial Z_j(1, Fo)}{\partial X}\right\} = (\vartheta_2^2-\vartheta_1^2)\left[1-(1-\varepsilon)KoLu\frac{Bi_m}{Bi_q}\right] \tag{17A}$$

$$Pn\sum_{j=1}^{2}(-1)^j\left\{Z_j(1, Fo)+\frac{\vartheta_{3-j}^2}{Bi_m}\frac{\partial Z_j(1, Fo)}{\partial X}\right\} = (\vartheta_2^2-\vartheta_1^2). \tag{18A}$$

The obtained problem (14A)–(18A) is a part of the general one-dimensional case {47}–{50}, whose solution {54} for the case considered, has the form:

$$Z_j(X, Fo) = Z_j^0(X)-\sum_{i=1}^{\infty}G_i\frac{g(0)}{\mu_i^2}\psi_{j,i}(X)\,e^{-\mu_i^2 Fo} \tag{19A}$$

where $\psi_{j,i}(X)$ is defined by equations {51}–{53}, G_i by equation {55}, $g(0)$ by equation {56} and $Z_j^0(X)$ by equations {57}–{59}.

Equations {51}–{53}, generating the eigenfunctions and eigenvalues, take the form:

$$\psi_j''(X)+\mu^2\vartheta_j^2\psi_j(X) = 0, \quad j = 1, 2 \tag{20A}$$

$$\psi_j'(0) = 0, \quad j = 1, 2 \tag{21A}$$

$$\sum_{j=1}^{2}(-1)^j(1-\vartheta_{3-j}^2) \times\left\{\left[1+(1-\vartheta_j^2)\frac{1-\varepsilon}{\varepsilon}Lu\frac{Bi_m}{Bi_q}\right]\psi_j(1)+\frac{1}{Bi_q}\psi_j'(1)\right\} = 0 \tag{22A}$$

$$\sum_{j=1}^{2}(-1)^j\left\{\psi_j(1)+\frac{\vartheta_{3-j}^2}{Bi_m}\psi_j'(1)\right\} = 0. \tag{23A}$$

The solution of the system (20A) is

$$\psi_j(X) = C_j\cos(\vartheta_j\mu X)+D_j\sin(\vartheta_j\mu X) \tag{24A}$$

where C_j and D_j are constants, which are to be determined. From the boundary conditions (21A) it follows that $D_j = 0$. Substitution of (24A) into (22A) and (23A), and having in mind that $\vartheta_j^2\vartheta_{3-j}^2 = 1/Lu$, gives:

$$\sum_{j=1}^{2}(-1)^j(1-\vartheta_{3-j}^2)a_j(\mu)C_j = 0 \tag{25A}$$

$$\sum_{j=1}^{2}(-1)^j b_j(\mu)C_j = 0 \tag{26A}$$

where

$$a_j(\mu) = \left[1+(1-\vartheta_j^2)\frac{1-\varepsilon}{\varepsilon}Lu\frac{Bi_m}{Bi_q}\right] \times\cos(\vartheta_j\mu)-\frac{\vartheta_j\mu}{Bi_q}\sin(\vartheta_j\mu) \tag{27A}$$

$$b_j(\mu) = \cos(\vartheta_j\mu)-\frac{\mu}{LuBi_m\vartheta_j}\sin(\vartheta_j\mu). \tag{28A}$$

The system (25A)–(26A) has C_j as a non-zero solution when:

$$(1-\vartheta_2^2)a_1(\mu)b_2(\mu)-(1-\vartheta_1^2)a_2(\mu)b_1(\mu) = 0. \tag{29A}$$

The obtained characteristic equation {29} defines an infinite series of eigenvalues

$$\mu_1 < \mu_2 < \ldots < \mu_i < \ldots.$$

From equation {12} and using (10A), one gets:

$$\sigma_{3-j} = (-1)^{j+1}\frac{Pn}{\vartheta_{3-j}^2}c_j \quad (30A)$$

where

$$c_j = (\vartheta_j^2-1)\left(\frac{1}{Bi_q}-\frac{1}{LuBi_m}\right)-\frac{KoPn}{Bi_q} \quad (31A)$$

It follows then from equation {55} for G_i that

$$G_i = -\frac{2b_2^2(\mu_i)}{PnC_1^2}\{b_2^2(\mu_i)c_2d_1(\mu_i)-b_1^2(\mu_i)c_1d_2(\mu_i)\}^{-1} \quad (32A)$$

where

$$d_j(\mu_i) = 1+\frac{\sin(\vartheta_j\mu_i)}{\vartheta_j\mu_i}\cos(\vartheta_j\mu_i). \quad (33A)$$

After some simple mathematical transformations of {56}, one gets:

$$g(0) = -C_1\frac{Pn(\vartheta_2^2-\vartheta_1^2)}{b_2(\mu_i)} \times\left\{\frac{Ki_q}{\vartheta_2^2-\vartheta_1^2}[c_2b_2(\mu_i)-c_1b_1(\mu_i)] +\left[\left[1-(1-\varepsilon)KoLu\frac{Bi_m}{Bi_q}\right]b_1(\mu_i) +\frac{\vartheta_2^2-1}{Pn}a_1(\mu_i)\right]b_2(\mu_i)\right\}. \quad (34A)$$

After substitution of (24A), (32A) and (33A) in the solutions (19A) and having in mind (26A), one obtains

$$Z_j(X,Fo) = Z_j^0(X)-(\vartheta_2^2-\vartheta_1^2)\sum_{i=1}^{\infty}A_ib_{3-j}(\mu_i)\cos(\vartheta_j\mu_iX)e^{-\mu_i^2Fo} \quad (35A)$$

where

$$A_i = \frac{2}{\mu_i^2}\left\{\frac{Ki_q}{\vartheta_2^2-\vartheta_1^2}[b_2(\mu_i)c_2-b_1(\mu_i)c_1] +\left[\left[1-(1-\varepsilon)KoLu\frac{Bi_m}{Bi_q}\right]b_1(\mu_i) +\frac{\vartheta_2^2-1}{Pn}a_1(\mu_i)\right]b_2(\mu_i)\right\} \times\{b_2^2(\mu_i)c_2d_1(\mu_i)-b_1^2(\mu_i)c_1d_2(\mu_i)\}^{-1}. \quad (36A)$$

The quasistationary solution $Z_j^0(X)$ is obtained through direct tackling of equation {57} under conditions {58}–{59} and for the case considered, it has the form

$$Z_j^0(X) = 1+Ki_q\left(1+\frac{1}{Bi_q}\right)-(1-\vartheta_j^2)\left(\frac{1}{Pn}+Ki_q\right)-Ki_q\vartheta_j^2X. \quad (37A)$$

After substitution of (36A) in (35A) are obtained the required potentials $Z_j(X,Fo)$ and hence from equation (11A) and (12A) the final solution of the problem:

$$T(X,Fo) = 1+Ki_q\left(1+\frac{1}{Bi_q}-X\right)-\sum_{i=1}^{\infty}A_ie^{-\mu_i^2Fo} \times\sum_{i=1}^{2}(-1)^j(\vartheta_j^2-1)b_j(\mu_i)\cos(\vartheta_{3-j}\mu_iX) \quad (38A)$$

$$\theta(X,Fo) = 1+PnKi_q(1-X)+Pn\sum_{i=1}^{\infty}A_ie^{-\mu_i^2Fo} \times(-1)^jb_j(\mu_i)\cos(\vartheta_{3-j}\mu_iX). \quad (39A)$$

APPENDIX B

Let us find the solution of Luikov's system

$$\frac{\partial T(X,Fo)}{\partial Fo} = \frac{\partial^2T(X,Fo)}{\partial X^2}-\varepsilon Ko\frac{\partial\theta(X,Fo)}{\partial Fo} \quad (1B)$$

$$\frac{\partial\theta(X,Fo)}{\partial Fo} = Lu\frac{\partial^2\theta(X,Fo)}{\partial X^2}-LuPn\frac{\partial^2T(X,Fo)}{\partial X^2},$$
$$0\leqslant X<\infty,\quad Fo>0 \quad (2B)$$

under the following boundary conditions

$$T(X,0)=0,\qquad \theta(X,0)=0 \quad (3B)$$

$$\frac{\partial T(0,Fo)}{\partial X} = -Ki_q \quad (4B)$$

$$\frac{\partial\theta(0,Fo)}{\partial X} = -PnKi_q \quad (5B)$$

$$T(\infty,Fo)\neq\infty,\qquad \theta(\infty,Fo)\neq\infty. \quad (6B)$$

In [2] after applying Laplace's transform to equations (1B) and (2B) and having in mind the initial condition (3B), the following image of the solution is obtained:

$$\bar{T}(X,s) = A_1\exp(\vartheta_1X\sqrt{s})+A_2\exp(\vartheta_2X\sqrt{s}) +A_3\exp(-\vartheta_1X\sqrt{s})+A_4\exp(-\vartheta_2X\sqrt{s}) \quad (7B)$$

$$\bar{\theta}(X,s) = \frac{1}{\varepsilon Ko}[A_1(\vartheta_1^2-1)\exp(\vartheta_1X\sqrt{s})+A_2(\vartheta_2^2-1) \times\exp(\vartheta_2X\sqrt{s})+A_3(\vartheta_1^2-1)\exp(-\vartheta_1X\sqrt{s}) +A_4(\vartheta_2^2-1)\exp(-\vartheta_2X\sqrt{s})] \quad (8B)$$

where

$$\vartheta_j^2 = \frac{1}{2}\left\{\left(1+\varepsilon KoPn+\frac{1}{Lu}\right)+(-1)^j \times\sqrt{\left[\left(1+\varepsilon KoPn+\frac{1}{Lu}\right)^2-\frac{4}{Lu}\right]}\right\}. \quad (9B)$$

From the boundary conditions are determined the constants A_1, A_2, A_3 and A_4. Hence the solutions (7B) and (8B) take the form:

$$\bar{T}(X,s) = \frac{Ki_q}{\vartheta_2^2-\vartheta_1^2}\left[\vartheta_1(\vartheta_2^2-1)\frac{\exp(-\vartheta_1X\sqrt{s})}{s\sqrt{s}} -\vartheta_2(\vartheta_1^2-1)\frac{\exp(-\vartheta_2X\sqrt{s})}{s\sqrt{s}}\right]. \quad (10B)$$

$$\bar{\theta}(X,s) = \frac{PnKi_q}{\vartheta_2^2-\vartheta_1^2}\left[-\vartheta_1\frac{\exp(-\vartheta_1X\sqrt{s})}{s\sqrt{s}} +\vartheta_2\frac{\exp(-\vartheta_2X\sqrt{s})}{s\sqrt{s}}\right]. \quad (11B)$$

The inverse Laplace transform of (10B) and (11B) gives

$$T(X,Fo) = \frac{2(Fo)^{\frac{1}{2}}Ki_q}{\vartheta_2^2-\vartheta_1^2}\left\{\frac{1}{\pi}\left[\vartheta_1(\vartheta_2^2-1)\exp\left(-\left[\frac{\vartheta_1 X}{2(Fo)^{\frac{1}{2}}}\right]^2\right) - \vartheta_2(\vartheta_1^2-1)\exp\left(-\left[\frac{\vartheta_2 X}{2(Fo)^{\frac{1}{2}}}\right]^2\right)\right] - \frac{X}{2(Fo)^{\frac{1}{2}}}\left[\left(\frac{1}{Lu}-\vartheta_1^2\right)\operatorname{erfc}\left(\frac{\vartheta_1 X}{2(Fo)^{\frac{1}{2}}}\right) - \left(\frac{1}{Lu}-\vartheta_2^2\right)\operatorname{erfc}\left(\frac{\vartheta_2 X}{2(Fo)^{\frac{1}{2}}}\right)\right]\right\} \quad (12B)$$

$$\theta(X,Fo) = Pn\frac{2(Fo)^{\frac{1}{2}}Ki_q}{\vartheta_2^2-\vartheta_1^2}\left\{\frac{1}{\pi}\left[-\vartheta_1\exp\left(-\left[\frac{\vartheta_1 X}{2(Fo)^{\frac{1}{2}}}\right]^2\right) + \vartheta_2\exp\left(-\left[\frac{\vartheta_2 X}{2(Fo)^{\frac{1}{2}}}\right]^2\right)\right] - \frac{X}{2(Fo)^{\frac{1}{2}}}\left[-\vartheta_1^2\operatorname{erfc}\left(\frac{\vartheta_1 X}{2(Fo)^{\frac{1}{2}}}\right) + \vartheta_2^2\operatorname{erfc}\left(\frac{\vartheta_2 X}{2(Fo)^{\frac{1}{2}}}\right)\right]\right\}. \quad (13B)$$

DISTRIBUTIONS DE TEMPERATURE ET D'HUMIDITE PENDANT LE SECHAGE D'UNE COUCHE DE MATERIAU HUMIDE

Résumé—En employant le système d'équations différentielles obtenues par Luikov, on étudie le séchage d'une couche de matériau en contact avec une plaque chaude. Dans cet article est traité le problème déjà considéré par Bruin [1] avec l'hypothèse simplifiée, de l'influence négligeable du gradient de potentiel d'humidité sur le transport de l'humidité. L'analyse présentée ici est basée sur la solution analytique exacte. L'influence des paramètres sans dimension sur les distributions de température et de potentiel d'humidité est illustrée par des exemples numériques.

TEMPERATUR- UND FEUCHTIGKEITSVERTEILUNG BEI KONTAKTTROCKNUNG EINER FEUCHTEN PORIGEN SCHICHT

Zusammenfassung—Mit Hilfe des Luikovschen Differentialgleichungssystems wird die Trocknung einer Schicht feuchten Materials, die sich in Kontakt mit einer Heizplatte befindet, untersucht. In dieser Zeitschrift wurde dieselbe Aufgabe von Bruin [1] mit der Vereinfachung betrachtet, daß die Feuchtigkeitsbewegung unter dem Einfluss des Druckgradients vernachlässigbar klein ist. In der vorliegenden Arbeit wird die genaue analytische Lösung zum Analysieren des Problems, ohne die obenerwähnten Einschränkungen, angewandt. Der Einfluß dimensionsloser Parameter auf Temperatur- und Feuchtigkeitsverteilung wird durch numerische Beispiele gezeigt.

РАСПРЕДЕЛЕНИЕ ТЕМПЕРАТУРЫ И ВЛАГОСОДЕРЖАНИЯ ПРИ КОНТАКТНОЙ СУШКЕ ВЛАЖНОГО ПОРИСТОГО СЛОЯ

Аннотация — С помощью системы дифференциальных уравнений Лыкова исследована сушка слоя влажного материала, находящегося в контакте с горячей пластиной. Эта же задача решалась Бруином [1] при упрощающем допущении о том, что перенос влаги под влиянием градиента потенциала влагопереноса пренебрежимо мал. Данный анализ проведен на основе точного аналитического решения без упомянутого ограничения. Влияние безразмерных параметров на распределение потенциалов температуры и влагосодержания иллюстрируется численными примерами.